I will explain a new method for studying Hamilton-Jacobi PDE with nonconvex Hamiltonians. The new idea is to introduce appropriate solutions to the adjoint of the formal linearization: this leads to new integration-by-parts techniques that augment the standard maximum principle tricks.