Stanford Department of Mathematics Colloquium

The nearest way: from lines to geometry

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Abstract

Since the early days of the calculus of variations, it has been understood that the idea of lines as shortest curves generalizes to geodesics on surfaces in a natural way, and, in 1854, B. Riemann famously made the leap to the understanding that, in a wide variety of situations, a geometric variational principle could explain the observed phenomenon of geodesics in many abstract settings, ultimately paving the way for Einstein's Theory of General Relativity.

However, since the underlying 'geometric field' is not directly observable, this raises the interesting question of whether one can recover the geometric structure from the knowledge of its geodesics. (For example, could one have deduced the structure of General Relativity's metric tensor by observing the motions of the planets, as Newton is said to have deduced the inverse square Law of Universal Gravitation from Kepler's Laws.)

Surprisingly, while this 'inverse problem' has been studied for almost as long as the calculus of variations itself, with important contributions having been made by Jesse Douglas in the 1930s (among others), there remain curious gaps in our understanding. For example, the problem, even on surfaces, of determining when a family of putative geodesics are actually the geodesics of a metric (and how many such metrics there might be) was only solved recently.

This talk will survey the history outlined above and describe the solution of the surface geodesic problem. Little will be required of the audience beyond a basic understanding of the calculus of variations.

Thursday, May 28
4:15 p.m.
Room 380-W