Stanford Department of Mathematics Colloquium

STABILITY OF STRONG VISCOUS SHOCK LAYERS IN AN IDEAL GAS

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Abstract

By a combination of asymptotic ODE estimates and numerical Evans-function computations, we examine the spectral stability of shock-wave solutions of the compressible Navier–Stokes equations with ideal gas equation of state, for arbitrary strength shock waves.
Our main results are that, in appropriately rescaled coordinates, the Evans function associated with the linearized operator L about the wave, an analytic function analogous to the characteristic polynomial whose zeros correspond to eigenvalues of L, (i) converges in the strong shock limit to the Evans function for a limiting shock profile of the same equations, for which internal energy vanishes at one endstate; and (ii) has no unstable (positive real part) zeros outside a uniform ball. Thus, the rescaled eigenvalue ODE for the set of all shock waves, augmented with the (nonphysical) limiting case, form a compact family of boundary-value problems that can be conveniently investigated numerically. An extensive numerical study then yields unconditional stability, independent of amplitude for a range of parameter values including all common gases.
Besides its physical interest, we believe that this analysis has interest as an example where it is possible to carry out a rigorous global stability analysis by numerical techniques, the obvious obstacle being the need to treat an unbounded parameter range using finitely many operations.

Thursday, January 10
4:15 p.m.
Room 380-F