PASSIVE SCALAR EQUATION FOR KRAICHNAN TYPE VELOCITY FIELD

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Abstract

Propagation of a passive scalar (e.g. temperature) by a fluid flow with velocity field $v(t, x)$ is described by the transport equation

$$\frac{\partial \theta(t, x)}{\partial t} = -v(t, x) \cdot \nabla \theta(t, x), \quad t > 0;$$

$$\theta(0, x) = \theta_0(x).$$

If the velocity field is reasonably regular (e.g. Lipschitz continuous in $x$) the solution of the transport equation is given by

$$\theta(t, x) = \theta_0(X^{t_0}_{t, x}), \quad \text{where } X \text{ is the backward flow related to the velocity field } v,$$

$$dX^{t_0}_{t, x} = v(s, X^{t_0}_{s, x}), \quad t > s, \quad X^{t_0}_{t, x} = x.$$

In the main statistical models of turbulence such as the Kolmogorov and Kraichnan models, the velocity field is not Lipschitz continuous. In this case formula (1) does not hold anymore. In Kraichnan’s model which will be considered in this talk, the velocity $v$ is a statistically homogeneous, isotropic, and stationary Gaussian vector field with zero mean and covariance

$$E(v^i(t, x)v^j(s, y)) = \delta(t - s)C^{ij}(x - y),$$

where $C^{ij}(x) \sim C^{ij}(0)(1 - |x|^\gamma), \quad 0 < \gamma < 2$ for $|x| \ll 1$ and decays fast for $x$. The velocity field for this covariance is Hölder continuous (in $x$) with the exponent $\gamma/2$.

A stochastic flow corresponding to this velocity will be constructed and a Lagrangian representation of the solution of the transport equation extending formula (1) to Kraichnan...
velocity field will be presented. It will be shown that the flow corresponding to Kraichnan velocity is super-unstable which leads to the dissipation of energy of the passive scalar. Moreover, it will be shown that the dissipation of energy takes place if and only if the flow is super unstable.

Surprisingly, the solution of the transport equation driven by the Kraichnan velocity field is pathwise unique and square integrable as in the classical setting. It will be shown that the solution of this equations remains to be well defined and unique even for the velocity fields much less regular then the Kraichnan velocity.

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