Stanford Algebraic Geometry — Seminar —

LIMITS OF VARIETIES: SUBSCHEMES VS. BRANCHVARIETIES

ALLEN KNUTSON
San Diego

Abstract

Let F be a family of algebraic varieties in projective space, depending on a nonzero parameter t. One can define the $t = 0$ limiting subvariety, but this behaves badly (e.g. two points colliding to become one). The usual workaround is to introduce subschemes of projective space, whose coordinate rings may contain nilpotents. Then a nice limit exists uniquely, and there is a complete moduli space of subschemes, the Hilbert scheme.
In this talk I'll give an alternate approach, where one keeps "variety" and gives up "sub". Define a branchvariety of projective space as a reduced scheme with a finite map to projective space (i.e. proper with finite fibers). Then after introducing a high-enoughth root of t, the limit branchvariety exists uniquely, and there is a complete moduli stack of branchvarieties. Hartshorne's thesis is that the only locally constant invariant of families of subschemes is the Hilbert polynomial. In some sense, this is a negative result: to study families of subvarieties, one must buy a huge package deal of subschemes. There turn out to be many more locally constant invariants of branchvarieties.
This work is joint with Valery Alexeev of the University of Georgia.

Friday, January 26
3:15 p.m.
Room 383-N