

— Stanford Algebraic Geometry — — and Number Theory Seminar —

LIFTING CM ABELIAN VARIETIES TO CHARACTERISTIC 0

BRIAN CONRAD
University of Michigan

Abstract

An interesting consequence of Tate's work on abelian varieties over finite fields is that any such abelian variety can be lifted to an abelian scheme with complex multiplication over a local integer ring after two operations are applied: replacing the finite field with a finite extension and then passing to an isogenous abelian variety. It is natural to ask if these two operations are necessary to ensure the existence of a CM lifting over a local domain of characteristic 0. A decade ago, Oort gave counterexamples if an isogeny is not permitted (but extension of the finite field is allowed). What if we permit isogenies before lifting but we do not allow extension of the finite field? There are two versions of the question, depending on whether or not we require the lifting to be over a local normal domain. In the absence of a normality hypothesis there is always an affirmative answer (by deformation theory), but remarkably if we insist on normality then there is an arithmetic obstruction. We explain the arithmetic obstruction in some examples and then we discuss the proof that the arithmetic obstruction is the only obstruction for making CM lifts (up to isogeny) over a normal local domain without increasing the finite field. This is joint work with C-L. Chai and F. Oort.

Friday, March 23

3:15 p.m.
Room 383-N

<http://math.stanford.edu/ag/s0607/>