Stanford Algebraic Geometry — Seminar —

MURPHY'S LAW IN ALGEBRAIC GEOMETRY: BADLY-BEHAVED DEFORMATION SPACES

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Abstract

We consider the question: "How bad can the deformation space of an object be?" (Alternatively: "What singularities can appear on a moduli space?") The answer seems to be: "Unless there is some a priori reason otherwise, the deformation space can be arbitrarily ugly." Hence many of the most important moduli spaces in algebraic geometry are arbitrarily singular, justifying a philosophy of Mumford.

More precisely, every singularity of finite type over $\mathbb{Z}$ (up to smooth parameters) appears on the Hilbert scheme of curves in projective space, and the moduli spaces of: smooth projective general-type surfaces (or higher-dimensional varieties), stable maps, plane curves with nodes and cusps, stable sheaves, isolated threefold singularities, and more. The objects themselves are not pathological, and are in fact as nice as can be: the curves are smooth, the surfaces have very ample canonical bundle, the stable sheaves are torsion of rank 1, the singularities are normal and Cohen-Macaulay, etc.

Thus one can construct a smooth curve in projective space whose deformation space has any given number of components, each with any given singularity type, with any given non-reduced behavior along various associated subschemes. Similarly one can give a surface over $\mathbb{F}_p$ that lifts to p^7 but not p^8. (Of course the results hold in the holomorphic category as well.)

This talk is intended to be relatively expository. For details see math.AG/0411469 (Invent. Math., to appear) or my webpage.

Friday, October 14
3:15 p.m.
Room 383-N

http://math.stanford.edu/~vakil/s0506/