Stanford Algebraic Geometry — Seminar —

DRINFELD MODULAR VARIETIES AND CURVES WITH MANY RATIONAL POINTS

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Abstract

Almost 60 years ago Andre Weil proved a formula for the number of rational points on a smooth projective algebraic curve C of genus g over a finite field $\mathbb{F}_q$. This formula, known as the Riemann hypothesis for curves, provides an upper bound on the maximum number of rational points possible:

$$\#C(\mathbb{F}_q) \leq q + 1 + 2g\sqrt{q}.$$

There are many cases in which Weil's bound cannot be attained. In fact, when g is large compared to q, this inequality can be improved significantly. Let $N_q(g)$ be the maximum number of $\mathbb{F}_q$-rational points on any curve over $\mathbb{F}_q$ of genus g. The Weil bound implies that for q fixed,

$$\lim_{g\to\infty} N_q(g)/g \leq 2\sqrt{q},$$

but, as Drinfeld and Vladut proved, this bound can be improved to $\sqrt{q} - 1$. In order to test the sharpness of the Drinfeld-Vladut bound, it is necessary to produce curves with many points. There are different approaches to this question. Manin and Vladut used supersingular points on Drinfeld modular curves to show that the Drinfeld-Vladut bound is in fact sharp when q is a square.

In my talk, which will be mostly expository, I shall explain Manin-Vladut construction, and indicate a possible application of higher dimensional Drinfeld modular varieties to the same question.

Friday, November 4
3:15 p.m.
Room 383-N