Stanford Algebraic Geometry
— Seminar —

THE UNEXPECTED POWER OF THE RANK 6 PLÜCKER RELATION: A NEW VIEW OF GRASSMANNIANS FROM MATHEMATICAL PHYSICS

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Abstract

Grassmannians are algebraic projective varieties defined by the quadratic polynomial equations called "the Plücker relations". They arise in many areas of mathematics in part because they geometrically parametrize the set of subspaces of a fixed dimension. Moreover, they have been found to play an important role in nonlinear dynamics and quantum field theory due to Sato's theorem which shows that many of the differential and difference equations in mathematical physics are merely disguised versions of the Plücker relations. Generally, this correspondence is applied in only one direction: using our knowledge of Grassmannians to solve these functional equations. However, in this talk I will describe a result that goes in the other direction. Inspired by the special role played by the simplest of these difference equations, I conjectured that the simplest Plücker relation (having quadratic rank 6) is in some sense more powerful than all of the others. The result is a theorem (proved this past summer with two undergraduates here at the College of Charleston) which offers an alternative set of polynomials to define the Grassmannian variety. We can replace the Plücker relations with a smaller set of simpler (in the sense that they all have rank 6) polynomial relations. Moreover, since these other polynomials are just the pull-backs of the one Plücker relation for $Gr(2,4)$ under linear maps from the space of k-forms, we can say that the *only* Plücker relation you need is this extremely simple one.

Tuesday, February 7
12:30 p.m. **Note unusual date and time!**
Room 383-N

http://math.stanford.edu/~vakil/s0506/