Geometry and PDE

A conference in honor of Leon Simon's 60th birthday

Stanford University
September 15–18, 2005

Schedule

Thursday, September 15

12:00–1:15

Refreshments and Registration

1:15–2:15

John Hutchinson—The Australian National University

TITLE: *Fractal sets and measures with a controlled degree of variability*

ABSTRACT: Many fractals (sets and measures) can be generated in a natural manner by using Iterated Function Systems [IFSs]. An IFS F is collection of functions ($f_1, \ldots, f_N$) with corresponding weights or probabilities ($w_1, \ldots, w_N$). Fractals generated in this manner are typically self similar, in the sense that the "subfractals" at each level of magnification (think of observing under a microscope) are similar to the original fractal up to certain transformations (e.g., similitudes, affine or projective if all the f_n are similitudes, affine or projective, respectively). Their Hausdorff dimension is typically nonintegral and much work has been done on computing the dimension in various cases.

In a related manner, one can generate random fractals by choosing the IFS F at each "application" of construction according to some *a priori*, given probability distribution. (An example is given by Brownian paths.) In this context, fractals generated by a single IFS are then called deterministic.

I will discuss a new class of V-variable fractals, generated by a given set of 2 or more IFSs, and intermediate between deterministic fractals and realizations of random fractals. The integer parameter V refers to the fact that at each level of magnification all subfractals of a Vvariable fractal fall into, at most, V disjoint classes, up to certain transformations (e.g., affine or projective). It turns out that such fractals have some surprising properties. In particular, certain V-tuples of V-variable fractals are, in a natural sense, elements of a single deterministic "superfractal," and all V-variable fractals can be obtained in this manner. This leads to interesting results concerning the generation of V-variable fractals, analysis of their dimensions (it depends on V), applications to generating standard random fractals, and a number of other interesting properties.

This is joint work with Michael Barnsley and Orjan Stenflo.

2:15–2:30	Break
2:30–3:30	**Jean Taylor**—Rutgers University **Title:** *Crystal growth coupled to relative translation* **Abstract:** It has been found, by molecular dynamics calculations and experiments, that when one crystal grows into another, there is typically some coupling due to the atomic rearrangements resulting in a relative translation along the interface of one crystal with respect to the other. Sliding is also possible, especially at elevated temperatures. John Cahn and I have been working on continuum mathematical models for this behavior.
3:30–4:00	Break
4:00–5:00	**Fang Hua Lin**—Courant Institute/New York University **Title:** *Stable Stationary Maps into Spheres* **Abstract:** TBA
Friday, September 16	
9:00–9:30	Continental breakfast
9:30–10:30	**Tatiana Toro**—University of Washington **Title:** *A generalization of Reifenberg's theorem in* R^3 **Abstract:** Two dimensional minimal cones were fully classified by Jean Taylor in the mid '70s. In joint work with G. David and T. De Pauw, we prove that a closed set, which is close to a minimal cone at all scales and at all locations, is locally a bi-Hölder image of a minimal cone. This result is analogous to Reifenberg's disk theorem. A couple of applications will be discussed.
10:30–11:00	Break
11:00–12:00	**Robert Gulliver**—University of Minnesota **Title:** *Total curvature and isotopy of graphs in* R^3 **Abstract:**
12:00–1:15	Lunch break

1:15–2:15	**Robert Bartnik**—Monash University TITLE: *Second variation in general relativity* ABSTRACT: The formula for the second variation of area of a hypersurface plays a key role in several seemingly unrelated classical computations in general relativity. I'll describe some of these applications.
2:15–2:30	Break
2:30–3:30	**Bill Allard**—Duke University TITLE: *On the regularity and curvature properties of level sets of minimizers for denoising models using total variation regularization.* ABSTRACT: Let Ω be an open subset of $\mathbf{R}^n$ where $2 \leq n \leq 7$; the reason for restriction on n is that our work will make use of the regularity theory for area minimizing hypersurfaces. Let $s \in \mathbf{L}_1(\Omega) \cap \mathbf{L}_\infty(\Omega)$, let $\gamma : \mathbf{R} \to [0, \infty)$ be zero at zero, nondecreasing and smooth on $[0, \infty)$ and convex and, for $f \in \mathbf{L}_1(\Omega)$, let $$F(f) = \int_\Omega \gamma(f(x) - s(x)) d\mathcal{L}^n x;$$ $\mathcal{L}^n$ here is Lebesgue measure on $\mathbf{R}^n$. In the denoising literature, F would be called a fidelity term in that it measures deviation from s, which could be a noisy grayscale image. Let $\varepsilon > 0$ and, for $f \in \mathbf{L}_\infty(\Omega)$, let $$F_\varepsilon(f) = {}_\varepsilon\mathbf{TV}(f) + F(f);$$ where $\mathbf{TV}(f)$ is the total variation of f. A minimizer of F_ε is called a **total variation regularization** (**TVR**) of s. Rudin, Osher and Fatemi and Chan and Esedoglu have studied TVRs of F where $\gamma(y) = y^2$ and $\gamma(y) = y, y \in \mathbf{R}$, respectively. In applications, n is typically 2, 3 or 4 and f, as above, is thought to be a "denoising" of s. Let f be a TVR of s. The first main result of this work is that the reduced boundaries of the sets $\{f \geq y\}$, $y \in \mathbf{R}$ are $C^{1+\mu}$ hypersurfaces for any $\mu \in (0, 1)$ in case $n > 2$ and any $\mu \in (0, 1]$ in case $n = 2$; moreover, the generalized mean curvature of the sets $\{f \geq y\}$ will be bounded by constants one can readily determine from the essential supremum of $\|s\|$. In fact, this result holds for a rather general class of fidelities. A second result gives precise curvature information about the reduced boundary of $\{f \geq y\}$ near points where s is smooth. This curvature information will allow us to construct a number of interesting examples of TVRs. In addition to providing insight as to the nature of TVRs, these examples may be used to validate computational schemes which purport to approximate TVRs.
3:30–4:00	Break

4:00–5:00	**Joel Spruck**—Johns Hopkins University **Title:** *Remarks on constant mean curvature graphs in MXR* **Abstract:**
Saturday, September 17	
9:00–9:30	Continental breakfast
9:30–10:30	**Peter Topping**—University of Warwick **Title:** *Reverse bubbling type behavior for Ricci flows* **Abstract:** I will discuss an unconventional existence theorem for the Ricci flow which yields flows exhibiting behavior reminiscent of reverse bubbling in the harmonic map flow.
10:30–11:00	Break
11:00–12:00	**Claus Gerhardt**—University of Heidelberg **Title:** *Minkowski-type problems for convex hypersurfaces in the sphere* **Abstract:** We consider problems of the form $F=f(\nu)$ for strictly convex, closed hypersurfaces in $S^{\{n+1\}}$, and solve it for a class of curvature functions F, that includes the H_k, $1 \leq k \leq n$, and $\|A\|^2$, or more precisely, all F's the inverses of which are of class (K).
12:00–1:15	Lunch break
1:15–2:15	**Hyam Rubinstein**—University of Melbourne **Title:** *Minimal surfaces in hyperbolic 3-manifolds* **Abstract:** TBA
2:15–2:30	Break
2:30–3:30	**Mu-Tao Wang**—Columbia University **Title:** *A convergence result of the Lagrangian mean curvature flow* **Abstract:** We discuss the deformation of the graph of a symplectomorphism between Riemann surfaces by the mean curvature flow. In particular, a new integral estimate is derived and is applied to improve an earlier convergence result.
3:30–4:00	Break

4:00–5:00	**Neshan Wickramasekera**—University of California, San Diego **Title:** *Stable branched minimal hypersurfaces* **Abstract:** I will describe a regularity theorem for a certain class of stable minimal hypersurfaces, and some of its corollaries which include a compactness theorem for the hypersurfaces and a decomposition theorem for their singular sets.
6:30–9:30	**Banquet at the Faculty Club in honor of Leon Simon. A reception at 6:30, followed by dinner at 7:00. Spouses, family, and friends are all welcome.**
Sunday, September 18	
9:00–9:30	Continental breakfast
9:30–10:30	**Craig Evans**—University of California at Berkeley **Title:** *Aronsson's equation* **Abstract:** I will discuss some of the analytic properties of solutions to Aronsson's equation, an extremely degenerate elliptic PDE that comes up as an Euler-Lagrange equation in "calculus of variations in the sup-norm".
10:30–11:00	Break
11:00–12:00	**Neil Trudinger**—The Australian National University **Title:** *Monge-Ampere type equations and optimal transportation* **Abstract:** We will explain the connection between optimal transportation and Monge-Ampere type equations. In particular we will discuss conditions on the cost functions which ensure the existence of classical solutions.