Thm (W) The poset of cells of $(Gr_{k,n})_{\geq 0}$ is Eulerian — if you fix a cell C & look at all cells in closure, the alternating sum of cells is 1.

This + CW complex ⟹ The Euler Char. of the closure of each cell is 1.

Explicit Decription of face poset of P_G.

- Can assume that G is bipartite

Def A planar-perfect matching of a plabic graph G is a subset of edges M s.t. each internal vertex of G is incident to the edge in M.

Def An elementary subgraph of G is a collection of edges which obtained by taking a union of several planar-perfect matchings.

Prop The face lattice of P_G is isomorphic to the lattice of all elementary subgraph of G obtained by inclusion.

Prop (PSW) Suppose we have a map $\Psi : (\mathbb{R}_{>0})^n \longrightarrow \mathbb{P}^{N-1}$

$$(t_1, \dots, t_n) \longmapsto [h_1(t_1, \dots, t_n) : \dots : h_N(t_1 \dots t_n)]$$

where the h_i's are Laurent polynomials w. positive coeff.

Let S be the set of all exponent vectors in $\mathbb{Z}^n$ which occurring among the h_i's & let Q be the conv. hull of S. Then Ψ extends to a natural map

$$\Psi' : X_Q \longrightarrow \mathbb{P}^{N-1}$$

which is well-defined on $(X_Q)_{\geq 0}$.