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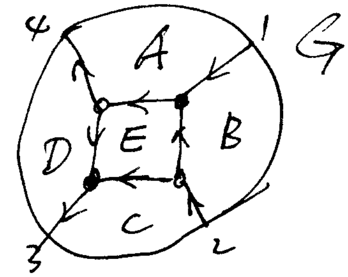
Theorem (W): The poset of cells of $(G/P)_{\geq 0}$ — in particular $(Gr_{kn})_{\geq 0}$ — is the poset of cells of a regular CW-complex homomorphic to a ball.

Def: A platic graph is a planer bicolored graph embedded in a disk.
internal
- Vatice are black or white
- Boundary vertices have deg 0 or 1



Def: A perfect orientation O of a platic graph is an orientation of the edge s.t. each black vertex has exactly one outgoing edge.
+ --- white ---------- ingoing edge.

Parametrization of cell
- Label faces of G with variables s.t. $ABCDE = 1$
- For each pair (i,j) consider all directed paths $i \to j$

$$\begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 1 & 0 & BCE & BCDE \\ 2 & 0 & 1 & C+CE & CDE \end{array} \longmapsto (1, C+CE, CDE, BCE, BCDE, BC^2DE^2)$$

This gives a parametrization of the cell is map
$$(\mathbb{R}_{>0})^4 \longrightarrow (Gr_{24})_{\geq 0}$$

Construction: Fix a platic graph cell a perfect orientation O.

Define a cone G_O as follows:
For each directed path in O we associate a half space.
In previous example:

$$C \geq 0, \quad C + E \geq 0$$
$$c + d + e \geq 0, \quad b + c + e \geq 0$$