

Matching polytopes, toric geometry, and the non-negative part of the Grassmannian.

Program

1. What is $(Gr_{kn})_{\geq 0}$?
2. Motivating results + conjecture
3. Parametrizations of cells
4. Building Toric varieties
5. Results

Grassmannian $Gr_{kn}(\mathbb{R}) = \{V \subset \mathbb{R}^n \mid \dim V = k\}$
 $= GL_n \setminus$

Totally non-neg part $(Gr_{kn})_{\geq 0} =$ subset of $Gr_{kn}(\mathbb{R})$
 s.t. all maximal minors positive

Ex $\begin{pmatrix} 1 & 0 & -1 & -2 \\ 0 & 1 & 3 & 9 \end{pmatrix} \mapsto (1, 3, 8, 1, 2, 2)$

Postnikov: \exists is a natural alg. cell decomposition of $(Gr_{kn})_{\geq 0}$.
 cells specified by saying which Plücker
 coords are $\neq 0$, which are 0

Parametrizations of cells by $(\mathbb{R}_{>0})^d$

(Rietz)

• $(Gr_{kn})_{\geq 0}$ is contractible (Lusztig)

• Conjecture: $(Gr_{kn})_{\geq 0}$ w. the natural cell decomposition is a regular CW complex homomorphic to a ball.

