CENTRE
DE RECHERCHES
MATHÉMATIQUES

Hugh Thomas
2007.5.28

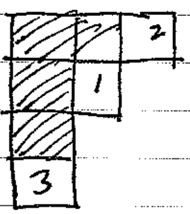
Tableau (Combinatorics of (K-) Dual Equivalence.

- jeu de taquin
- dual equivalence
- K-theoretic version
- minuscule extension

Partion λ

$\nu = (3,2,1,1)$ $\lambda = (2,1,1)$

ν/λ



increasing in rows & columns

std. filling $\{1, \dots, |\mu/\lambda|\}$

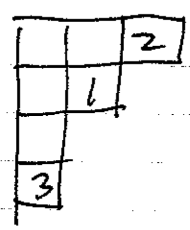
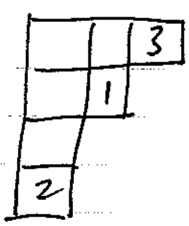
reflection apply jdt until none available
i.e., your shape is Ferres shape μ.

Fix T_μ std filling of μ

$C^{\nu}_{\lambda\mu}$ = # std filling shape ν/λ, rectifying to T_μ

- rectification is independent of order of slides
- # doesn't depend to T_μ

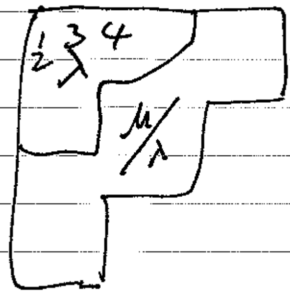
Give two fillings of shape ν/λ, dual equivalent iff any seq. of jdt moves preserves same shape.

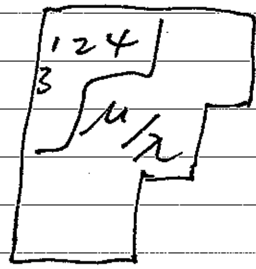
 

Main theorem of DE (Haiman) Any filling of same straight shape λ are dual equivalent.
↓ not skew

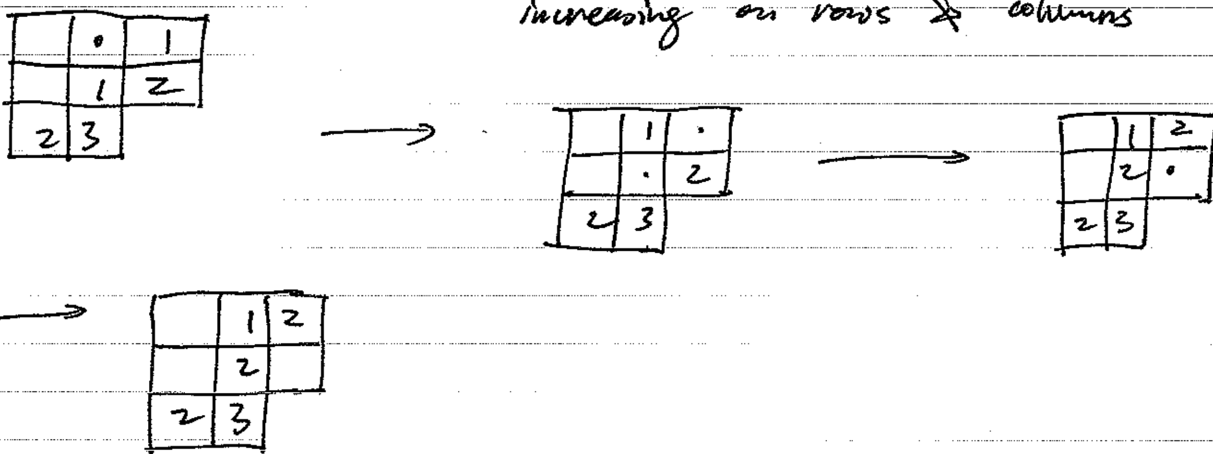
pf: $\begin{array}{|c|c|}\hline 1 & 2 \\ \hline 3 \\ \cline{1-1}\end{array} \underset{DE}{\equiv} \begin{array}{|c|c|}\hline 1 & 3 \\ \hline 2 \\ \cline{1-1}\end{array}$ induction

Consequences, (i) rectification is well-defined
(ii) # occurances T_μ independent of T_μ





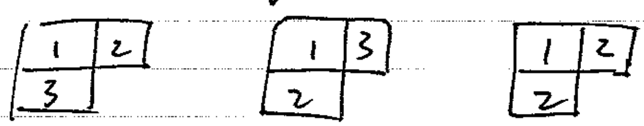
Increasing filling of ν/λ. filling using $1, \dots, q$
increasing on rows & columns

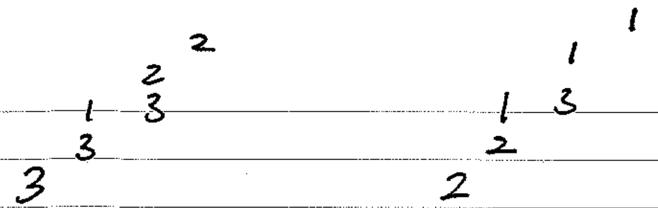


Choose set of inners.
consider filling on · 1's

in any component with · 1's, switch them.
do same for · & 2's, etc.

Main theorem of DE for increasing: skew shape
Any two increasing fillings of λ are D.E

pf: 



$C^{\nu}_{\lambda\mu}$ = # increasing fillings ν/λ rectifying to T_μ.

Thm $C^{\nu}_{\lambda\mu}(-1)^{|\nu|-|\lambda|-|\mu|}$ form structure constants for K-theory of Grassmannien.

Why?
(i) Cartan's gives manifesty S_3-symmetric rule.
(ii) Minuscule shape