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# Equivalent classes of matroid realization spaces

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$\underbrace{v_1, \dots, v_k}_{\mathcal{C}\text{ configration}} \in \mathbb{C}^n$

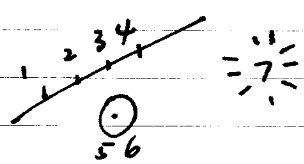
$$X_{\mathcal{C}} = \{ (u_1, \dots, u_k) \in (\mathbb{C}^n)^k \mid \dim \operatorname{span}(u_i)_{i \in I} = \dim \operatorname{span}(v_i)_{i \in I} \quad \forall I \subseteq \{1, \dots, k\} \}$$

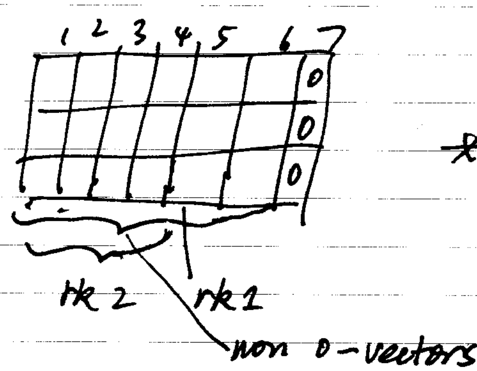
Observe: $X_{\mathcal{C}}$ doesn't change if we rescale $v_i \rightsquigarrow \lambda_i v_i$, $\lambda_i \in \mathbb{C}^*$

Hence, to define $X_{\mathcal{C}}$, it is enough to know

also configration $\rightarrow$ $\begin{cases} [v_i] \in \mathbb{P}^{n-1} \\ + \text{ a list of those } v_i = 0 \end{cases}$

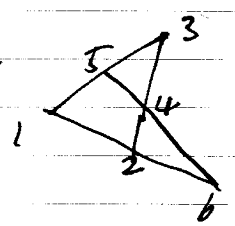
ex. $n=3$




$*$

ex.




Menelaus

$Y_{\mathcal{C}} = \overline{X}_{\mathcal{C}}$ matroid realization spaces

$\boxed{\text{Ideal of } Y_{\mathcal{C}}}$

$*$ $I = \Big\langle$ 3x3 minors, 2x2 minors, 1x1 minors