$$H_{H^i_{\underline{p}}(K[\Delta])}(s,t) = \sum_{F\in\Delta_W} \sum_{G\subset V} \dim_K \tilde{H}_{i-|F|-1}\left((\mathrm{lk}(F\cup G))_W ; K\right) \prod_{v_i\in G} \frac{s_i}{1-s_i} \prod_{w_j\in F} \frac{t_j^{-1}}{1-t_j^{-1}}$$

$s = (s_1, \dots, s_m)$

$t = (t_1, \dots, t_n)$

$m=0 \quad G=\phi, \qquad \prod_{v_i\in G}\frac{s_i}{1-s_i} = 1, \quad (\mathrm{lk}(F\cup G))_W = \mathrm{lk}\,F$

$$H_{H^i_{\underline{m}}(K[\Delta])}(t) = \sum_{F\in\Delta} \dim_K \tilde{H}_{i-|F|-1}(\mathrm{lk}(F); K) \prod_{w_j\in F} \frac{t_j^{-1}}{1-t_j^{-1}}$$

$$H^i_p(K[\Delta])_{(k,j)} = \bigoplus_{\substack{|a|=k,\ a\in\mathbb{Z}^m \\ |b|=j,\ b\in\mathbb{Z}^n}} H^i_p(K[\Delta])_{(a,b)}$$

__Prop__ $\dim_K H^i_p(K[\Delta])_{(k,j)} = \sum_{\substack{F\in\Delta_W \\ G\subset V}} d(i,F,G) \binom{k-1}{|G|-1} \binom{-j-1}{|F|-1}$

$$H^i_p(K[\Delta])_j = \bigoplus_k H^i_p(K[\Delta])_{(k,j)}$$

$j\in\mathbb{Z}$

$$\left(H^i_p(K[\Delta])_j\right)_k = H^i_p(K[\Delta])_{(k,j)}$$

if $\dim H^i_p(K[\Delta])_j = \text{constant}$ for $j << 0$

__Thm__

$$H_{H^i_p(K[\Delta])_j}(s) = \sum_{k=0}^{\infty} \dim_K \left(H^i_p(K[\Delta])_j\right)_k s^k$$

$$= \sum_{k=0}^{\infty} \dim_K H^i_p(K[\Delta])_{(k,j)} s^k$$

$$= \cdots = Q_j(s)/(1-s)^{m-c} \qquad Q_j(1)\neq 0$$