CENTRE
DE RECHERCHES
MATHÉMATIQUES

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Tameness of local cohomology of monomial ideals with respect to monomial prime ideals

R Noetherian graded ring $\quad H$ graded R-module

M_j for $j << 0$

M Artinian f.g. R-mod, $M_j = 0$ for $j << 0$.

Def: M is called tame, if there exists an integer j_0 s.t.

either $M_j = 0$ for $j \leq j_0$ or $M_j \neq 0$ for $j \leq j_0$.

$R_+ = \bigoplus_{i>0} R_i$, $\quad H^i_{R_+}(M)$

M f.g. R-module

Is the local cohomology modules $H^i_{R_+}(M)$ tame?

Brodmann & Hellus.

$H^i_{R_+}(M)_j$ is f.g. R_i-module and $H^i_{R_+}(M)_j = 0$ for $j >> 0$.

$\dim R_0 \leq 2$.

Cutkosky & Herzog, $\quad \dim R_0 = 3 \quad$ False
(C-H-C-S)

$M = k[y_1, \cdots, y_r]/I$, $\quad I$ monomial ideal $\quad$ true

Thm: Let $I \subseteq S = k[x_1, \ldots, x_m, y_1, \cdots, y_n]$ square-free.

$\mathbb{Z}^m \times \mathbb{Z}^n$-bigrading. $R = S/I \cong k[\Delta]$ for some simplicial complex Δ with vertices $\{v_1, \cdots, v_m, w_1, \cdots, w_n\}$.

$P = (y_1, \cdots, y_n)$