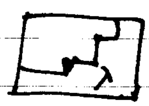
3) $\bar{A}_* (S_\lambda(F.)) = \bigoplus_{\mu \le \lambda} \mathbb{Z}[S_\mu(F.)]$

$S_\lambda = \sum_{\mu \le \lambda} 1\!\!1_{S_\mu^\circ}$, $\quad c_{SM}(S_\lambda) = \sum c_{SM}(S_\lambda^\circ)$

$$c_{SM}(S_\lambda^\circ) = \sum_{\mu \le \lambda} c(\lambda, \mu)[S_\mu]$$

GOAL : compute $c(\lambda, \mu)$

Bott – Samelson Resolutions

$\lambda =$  p

$n-p$

$$V(\lambda) = \left\{ S_1 \subset S_2 \subset \cdots \subset S_p : \begin{array}{l} \dim S_i = i \\ S_i \subset F_{\lambda_{p-i+1}+i} \end{array} \right\}$$

$$\pi_\lambda \downarrow \qquad (S_1 \subset \cdots \subset S_p)$$

$$S_\lambda(F.) \qquad \downarrow$$

FACTS 1) π_λ is an isom. over $S_\lambda^\circ(F.)$

2) $(\pi_\lambda)^{-1}(S_\lambda - S_\lambda^\circ) = \cup D_i$ – snc

$$c_{SM}(S_\lambda^\circ) = (\pi_\lambda)_* \left(\frac{c(TV(\lambda))}{\prod(1+D_i)} \cap [V(\lambda)] \right)$$

$[D_i] = x_i \in A^1(V(\lambda))$

$\lambda = (\lambda_1, \lambda_2)$ $\quad p = 2$

$$(*) \quad \frac{c(TV(\lambda))}{\prod(1+x_i)} = (1+x_1)^{\lambda_1 - \lambda_2} (1+x_2)^{\lambda_2 - 0} h_{\lambda_2}(1+x_1, x_2).$$

$\lambda = (\lambda_1, \ldots, \lambda_p)$ $\qquad (*) = \prod_{i=1}^{p} (1+x_i)^{\lambda_i - \lambda_{i+1}} \cdot h_{\lambda_{i+1}}(1+x_i, x_{i+1}, \ldots, x_p)$