THM (conj. D-G. MACPHERSON '74)

$\exists!$ natural transformation

$$C_*: F(X) \longrightarrow A_*(X) \quad \text{s.t.}$$

(1) If X smooth, $C_*(\mathbb{1}_X) = c(TX) \cap [X]$

(2)
$$\begin{array}{ccc} F(X) & \xrightarrow{C_*} & A_*(X) \\ {\scriptstyle f_*}\downarrow & & \downarrow{\scriptstyle f_*} \\ F(Y) & \xrightarrow{C_*} & A_*(Y) \end{array}$$

Def $\quad C_{SM}(X) = C_*(\mathbb{1}_X)$

$Y = pt \quad f_*(\mathbb{1}_X) = \chi(X).$

$$\int c_*(\mathbb{1}_X) = \chi(X) \qquad \mathbb{1}_X = \sum \mathbb{1}_{W_i}$$

Second Def of CSM classes

$X = \coprod W_i \qquad W_i$ is locally closed but smooth

$$\begin{array}{ccc} W \subset \overline{W} & \hookrightarrow & X \\ {\scriptstyle \pi}\uparrow & \nearrow{\scriptstyle \tilde{\pi}} & \\ \widetilde{W} & & \end{array}$$

$\pi^{-1}(\overline{W} \setminus W) = \bigcup D_i$ – SNC divisor
$= D$

$$C_{SM}(\mathbb{1}_W) = (\tilde{\pi})_* \left(c\left(\Omega^1_{\widetilde{W}}(\log D)^\vee\right) \cap [\widetilde{W}] \right)$$

$$0 \to \Omega^*_{\widetilde{W}} \to \Omega^1_{\widetilde{W}}(\log D) \longrightarrow \bigoplus \mathcal{O}_{D_i} \longrightarrow 0$$