CENTRE
DE RECHERCHES
MATHÉMATIQUES

Leonardo Constantin
Mihalcea
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Chern–Schwartz–MacPherson classes for Schubert cells in the Grassmannian

j. w. P. ALUFFI

$X = Gr(p, n)$

$$\begin{tikzcd} 0 \arrow[r] & S \arrow[r] \arrow[dr] & \mathbb{C}^n \arrow[r] \arrow[d] & Q \arrow[r] \arrow[dl] & 0 \\ & & X & & \end{tikzcd}$$

$TX = Hom(S, Q) \cong Q \otimes S^\vee$

$c(TX)$

i) $\int_X c(TX) = \int_X c_{top}(TX) = \chi(X)$ Poincaré–Hopf

ii) $c(TX) \cap [X] = \sum_{\mu \geq \lambda} \underset{\mathbb{Z}}{\underset{\cap}{d_{\lambda\mu}}} \cdot (\sigma_\lambda \sigma_\mu) \cap [X]$

$$d_{\lambda\mu} = \det \left(\binom{\lambda_i + p - i}{\mu_j + p - j} \right)_{1 \leq i, j \leq p} > 0$$

Question: How to generalize i) for singular varieties (maybe also ii))

II Def of CSM classes

$F(X)$: group of const. functions

$$F(X) = \left\{ \sum m_{Z_i} 1\!\!1_{Z_i} : \begin{array}{l} Z_i \subset X \text{ constructive} \\ m_{Z_i} \in \mathbb{Z} \end{array} \right\}$$

$f : X \longrightarrow Y$ proper

$f_* : F(X) \longrightarrow F(Y)$

$f_*(1\!\!1_Z) = \chi(f^{-1}(y) \cap Z)$