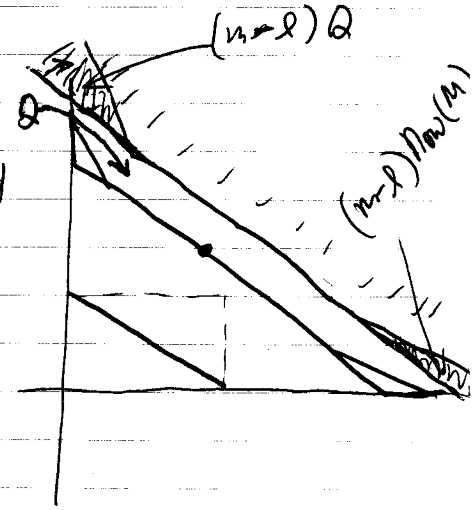
• $M \subset \langle t_i^{a_i} : i = 1, \cdots, n \rangle$

Then I is integrally closed iff M is int. closed

<u>Pf of Bagatelle</u> Assume $M = \overline{M}$

$\deg(t_1) = b, \quad \deg(t_2) = a$



I is finely $\mathbb{N}$-graded $\Rightarrow \overline{I}$ is graded

Pick a monomial in M of smallest degree

<u>THM</u> R Newton, $I \subset R$

$r \in \overline{I}$ iff $\exists \ell$ s.t. $(r^m) \in I^{m-\ell}, \ \forall m > \ell$

When does $t^u \in \overline{I}$?

$(t^u)^m \in I^{m-\ell} \quad \forall m > \ell \Rightarrow u \in (m-\ell)Q \quad \forall m > \ell$

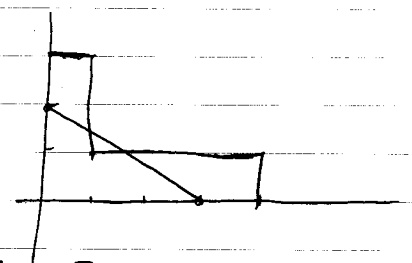
$\frac{m}{m-\ell} u \in Q \quad \forall m > \ell, \ u \in Q$

$I = \langle x^3 - y^2, x^4, y^3, xy \rangle$

$\langle x^4, y^3, xy \rangle$ is ~~not~~ closed

But

$\overline{I} = \langle x^3, y^2, xy \rangle$



<u>Example</u>:

$I = \langle x^3 + y^6, xy - y^3 \rangle + \underbrace{\langle x^4, x^5 y, x^2 y^3, x y^5, y^7 \rangle}_{M}$

$J = \langle x^3 - y^6, xy - y^3 \rangle + M$

$J = \overline{J}, \quad I \neq \overline{I} = I + \langle x^3 \rangle$