$(x-y)(z-w) \in \bar{I}$

$\bar{I} = I + \langle (x-y)(z-w) \rangle$ <u>not</u> a binomial

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<u>Q</u>: I is primary, is $\bar{I}$ primary? (Krull 1936) <u>NO</u> (Huneke 1986)

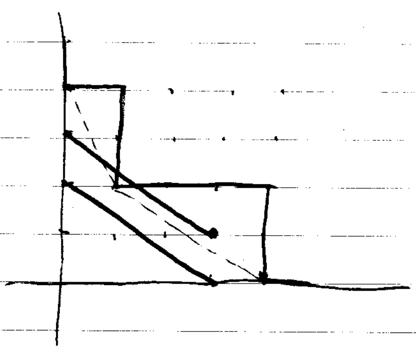
$I \subset k[\![x_1, x_2, x_3]\!]$, char $k = 2$

$I = \langle x^3 - y^2 z, x^4, xy^2, y^4 \rangle \xrightarrow{\langle x,y \rangle\text{-primary}} \subset k[x,y,z]$ char $k=0$

$\bar{I} = \langle x^3 - y^2 z, x^4, xy^2, y^4, x^3y, y^3 z \rangle$

$= \langle x^3 - y^2 z, x^4, x^3 y, xy^2, y^3 \rangle \cap \langle x^3 - y^2 z, x^4, xy^2, y^4, z \rangle$

<u>Example</u>: $I = \langle x^3 - y^2, x^4, xy^2, y^4 \rangle$



<u>Claim</u>: $\bar{I} = \langle x^3 - y^2 \quad x^4, xy^2, y^4, x^2 y, y^3 \rangle$

Bagatelle: $I \subset k[t_1, t_2]$. $I = \langle t_1^a - \lambda t_2^b \rangle + M$
where
- $\lambda \neq 0$
- $\gcd(a,b) = 1$
- $M \subset \langle t_1^a, t_2^b \rangle$

Then: I is integrally closed iff M is integrally closed.

<u>Generalized Bagatelle</u>

$I = \langle t_1^{a_1} - \lambda_1 t_2^{a_2}, t_2^{a_2} - \lambda_2 t_3^{a_3}, \dots, t_{n-1}^{a_{n-1}} - \lambda_{n-1} t_n^{a_n} \rangle + M$
- $\lambda_i \neq 0$
- $\mathrm{conv}\{a_i e_i\}$ has no lattice points except vertex