CENTRE
DE RECHERCHES
MATHÉMATIQUES

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Binomial Ideals & Integral Closure (w/ Milena Hering)

Def R ring, I ideal. An element $r \in R$ is integral over I if
(commutative) $r^m + a_1 r^{m-1} + \cdots + a_{m-1} r + a_m = 0 \qquad a_i \in I^i, \ i = 1, \cdots, m$

The integral closure of I

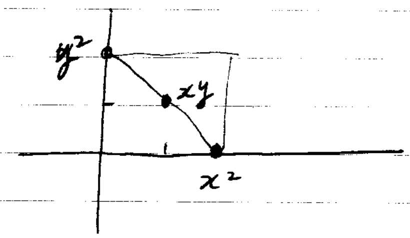
$$\bar{I} = \{ r \mid r \text{ integral over } I \}$$

If $I = \bar{I}$, I is integral closed.

- $I \subset \bar{I} \subset \sqrt{I}$
- $\bar{I}$ is an ideal, $\bar{\bar{I}} = \bar{I}$.

Example $k[X] = k[x_1, \cdots, x_n]$. M monomial ideal.

Define $\mathrm{New}(M) = \mathrm{conv}\, \{ \alpha \in \mathbb{N}^n \mid x^\alpha \in M \}$ Newton Polyhedron



Then

$$\bar{M} = \langle X^\beta \mid \beta \in \mathrm{New}(M) \rangle.$$

- $\bar{M}$ is a monomial ideal

THM If I is $\mathbb{Z}^d$-graded, so is $\bar{I}$.

$x^\beta \in \bar{M}$. Then $(X^\beta)^i$ appears with nozero coff. in $a_i \in M^i$

$(X^\beta)^i \in M^i$

M monomial ideal
- $\bar{M}$ a monomial ideal
- M P-primary, $\bar{M}$ is also P-primary.

How about I binomial?

Example $I = \langle x(x-y), y(x-y), z(z-w), w(z-w) \rangle$

$(x-y)^2 \in I \quad (z-w)^2 \in I \quad (x-y)(z-w) \in I^2$