This leads to a SS $(\bar{E}^r, \bar{d}^r)$ with

$$\bar{E}^0_{p,q} = \bigoplus_{\sigma \in \Sigma(d-p)} \Lambda^q E_\sigma \quad (= C_p(\Lambda^q \Sigma))$$

$$\bar{E}^1_{p,q} = H_p(\Lambda^q \Sigma) \Rightarrow H_p(X_\Sigma(\mathbb{R}))$$

$$\begin{array}{ccc} \sum \text{rank } \bar{E}^1_{p,q} & = & \sum \text{rank } E^2_{p,q} \\ \vee\!\!/ \ \| & & \vee\!/ \\ \sum b_i X_\Sigma(\mathbb{R}) & \overset{=}{\leq} & \sum b_j X_\Sigma(\mathbb{R}) \end{array}$$

Smith–Thom inequality

Thm: Δ reflexive Σ^* consists of smooth.

Then $(\bar{E}^r, \bar{d}^r)$ collapse at $\bar{E}^1$ and hence X_Σ is maximal