<u>Topology:</u> There exist spectral sequences (E^r, d^r), $(\bar{E}^r, \bar{d}^r)$ with

$$E^2_{p,q} \Rightarrow H_{p+q}(X_\Sigma(\mathbb{C}))$$
$$\bar{E}^1_{p,q} \Rightarrow H_p(X_\Sigma(\mathbb{R}))$$
$$E^2_{p,q} = \bar{E}^1_{p,q} = H_{pq}(\Sigma)$$

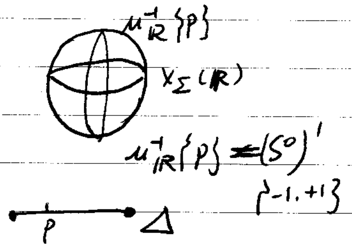
Use the moment map μ

$$\begin{tikzcd} X_\Sigma(\mathbb{C}) \arrow[dr, "\mu_{\mathbb{C}}"] & \\ \cup & \Delta \\ X_\Sigma \mathbb{R} \arrow[ur, "\mu_{\mathbb{R}}"'] & \end{tikzcd}$$

(E^r, d^r) is the $\mathbb{Z}_2$ Leray SS for $\mu_{\mathbb{C}}$.

$\mu_{\mathbb{R}}$: $p \in \operatorname{int} f$, $f < \Delta$, $\sigma \in \Sigma$ dual to f

$$\mu_{\mathbb{R}}^{-1}\{p\} = E_\sigma \; (= (S^0)^{\dim f})$$



This gives a cell structure on $X_\Sigma(\mathbb{R})$ with $2^{\dim f}$ copies of f, $f < \Delta$.

$$C_j(X_\Sigma(\mathbb{R})) = \bigoplus_{\sigma \in \Sigma(d-j)} H_0(E_\sigma)$$

Bitan-Franz - McGrory - van Hamel define a filtration of $H_0(E_\sigma)$ s.t.

$$Gr_* H_0(E_\sigma) = \Lambda^* E_\sigma \text{ naturally.}$$