The $\mathbb{Z}_2$ Hodge spaces of Σ are

$$H_{pq}(\Sigma) = H_p(\wedge^q \mathcal{E}) \qquad 0 \le q \le p \le d$$

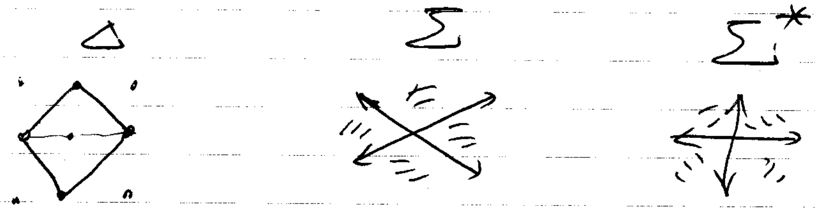
1) If Σ is a smooth fan, then $H_{pq}(\Sigma) = 0$ for $p \ne q$

$$\text{rank } H_{qq}(\Sigma) = h_q(\Delta)$$

<u>Def</u> A lattice polytope Δ is reflexive if $0 \in \text{int}\,\Delta$

and the polar polytope (dual) Δ^* (Δ°) is a lattice polytope

<u>notation</u>: Σ^* is normal fan of Δ^*
(= face fan of Δ)



<u>Theorem</u>: Assume the cones in Σ^* of $\dim \le e$ are smooth.
Then

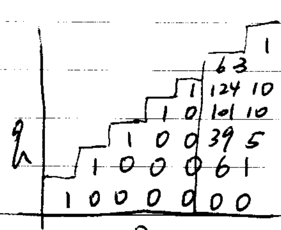
① $H_{pq}(\Sigma) = 0 \qquad q < p < e-1$

② $H_{qq}(\Sigma) = \mathbb{Z}_2 \qquad q < p-1.$

Let $\Delta = \text{conv}\{e_{i+1} - e_i \mid 1 \le i \le 6\}$ be the 6-dime crosspolytope.

Σ^* defines $\mathbb{P}^1 \times \mathbb{P}^1 \times \mathbb{P}^1 \times \mathbb{P}^1 \times \mathbb{P}^1 \times \mathbb{P}^1$

$e = d = 6$



$H_{pq}(\Sigma) = A_q(X_\Sigma, \mathbb{Z}_2)$
↑ is generated by closure of any q-torus orbit

$A_{d-2}(X_\Sigma, \mathbb{Z}_2) \longrightarrow H_{d-2}(X_\Sigma(\mathbb{R})) \quad q < e-1$