CENTRE
DE RECHERCHES
MATHÉMATIQUES

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$\mathbb{Z}/2\mathbb{Z}$ Hodge spaces of toric varieties

<u>Notation</u> $M, N \cong \mathbb{Z}^d$ are dual lattices

$\Sigma \subset N \otimes \mathbb{R}$ is the normal fan of a d-dim lattice polytope $\Delta \subset M \otimes \mathbb{R}$

<u>Def</u> A cosheaf $\mathcal{F}$ on Σ is a collection of $\mathbb{Z}_2$ vector spaces $(F_\sigma)_{\sigma \in \Sigma}$ with maps $\rho_{\tau,\sigma} : F_\sigma \longrightarrow F_\tau$ $\sigma < \tau$ s.t.

① $\sigma < \tau < \beta$ $\rho_{\beta,\tau}\rho_{\tau,\sigma} = \rho_{\beta,\sigma}$

② $\rho_{\sigma,\sigma}$ is the identity.

<u>Chain groups</u>: $C_p(\mathcal{F}) = \bigoplus_{\sigma \in \Sigma(d-p)} F_\sigma$ and $\partial_p : C_p(\mathcal{F}) \longrightarrow C_{p-1}(\mathcal{F})$

given by direct sum of maps

$$\sum \rho_{\tau,\sigma} : F_\sigma \longrightarrow \bigoplus_{\substack{\sigma < \tau \\ \tau \in \Sigma(d-p+1)}} F_\tau$$

$H_p(\mathcal{F})$ is the pth homology of the complex $(C_*(\mathcal{F}), \partial_*)$

<u>Note</u>: Coefficients are $\mathbb{Z}_2$.

$\mathcal{N}$ is defined via $\sigma \in \Sigma$, $N_\sigma = (\operatorname{span}\sigma \cap N)/(\operatorname{span}\sigma \cap 2N)$

rank is $\dim \sigma$.

Define $\mathcal{E}$ to be the cokernel of inclusion $\mathcal{N} \hookrightarrow N/2N$

$\frac{N}{2N}$ is the constant cosheaf.

E_σ has rank $= \operatorname{codim} \sigma$