Thm (Lenart) $i = \overset{i}{\boxed{}}$ $\quad C_{i,\mu}^{\nu} = \begin{cases} (-1)^{|\nu/\mu| - i} \cdot \binom{r(\nu/\mu) - 1}{|\nu/\mu| - i} & \text{if } \nu/\mu = \text{horizontal strip.} \\ 0 & \text{otherwise} \end{cases}$

(r = # rows)

Thm (B) $c_{\lambda\mu}^{\nu} = (-1)^{|\nu| - |\lambda| - |\mu|}$ # set-val tab.

Euler characteristics $\chi_X : K(X) \longrightarrow \mathbb{Z}$

$$[\mathcal{E}] = \sum_{i \geq 0} (-1)^i \dim H^i(X, \mathcal{E})$$

$\boxed{\text{Dual basis} \quad \mathcal{O}_\nu^* = (1 - \mathcal{O}_{\square}) \cdot \mathcal{O}_{\nu^\vee}}$

$$\Rightarrow \chi_X(\mathcal{O}_\lambda \cdot \mathcal{O}_\nu^*) = \delta_{\lambda,\nu}$$

Def $\alpha_1, \alpha_2, \alpha_3 \in K(X)$, set $\langle \alpha_1, \alpha_2, \alpha_3 \rangle_d$

$$= \chi_{M_d}(ev_1^*(\alpha_1) \cdot ev_2^*(\alpha_2) \cdot ev_3^*(\alpha_3))$$

Notice : $C_{\lambda\mu}^{\nu} = \chi_X(\mathcal{O}_\lambda \cdot \mathcal{O}_\mu \cdot \mathcal{O}_\nu^*)$

$$= \langle \mathcal{O}_\lambda, \mathcal{O}_\mu, \mathcal{O}_\nu^* \rangle_0$$

$$\mathcal{O}_\lambda * \mathcal{O}_\mu \cancel{=} \sum_{\nu, d} \langle \mathcal{O}_\lambda, \mathcal{O}_\mu, \mathcal{O}_\nu^* \rangle_d \, q^d \, \mathcal{O}_\nu$$

not associative

Quantum K-theory (Lee, Givental)

Def $N_{\lambda,\mu}^{\nu,0} = C_{\lambda\mu}^{\nu}$

$$N_{\lambda,\mu}^{\nu,d} = \langle \mathcal{O}_\lambda, \mathcal{O}_\mu, \mathcal{O}_\nu^* \rangle_d - \sum_{\kappa, 0 < e \leq d} N_{\lambda,\mu}^{\kappa, d-e} \cdot \langle \mathcal{O}_\kappa, \mathcal{O}_\nu^* \rangle_e$$