CENTRE
DE RECHERCHES
MATHÉMATIQUES

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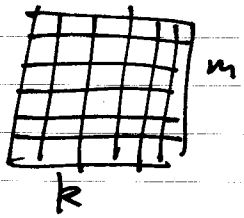
5.31

Quantum K-theory of Grassmannians w. Leonardo Mihalcea

$X = Gr(m, \mathbb{C}^n) = \{V \subseteq \mathbb{C}^n \mid \dim V = m\}$

$\dim X = mk$, $k = n-m$.

$\lambda = (\lambda_1 \geq \cdots \geq \lambda_m) =$ $\subseteq$ 

$X_\lambda = \{V \in X \mid \dim(V \cap \mathbb{C}^{k+1-\lambda_i}) \geq i,\ \forall\ 1 \leq i \leq m\}$

$\operatorname{codim} X_\lambda = |\lambda| = \sum \lambda_i$

$H^*(X) = \bigoplus_{\lambda \subseteq \boxed{m+k}} \mathbb{Z}[X_\lambda]$

Gromov-Witten invs.

A (rational) curve $C \subseteq X$ is image of $\mathbb{P}^1 \longrightarrow X$.

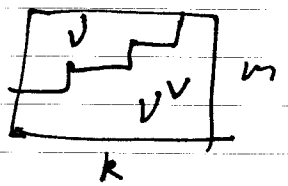
$\deg(C) = \#(C \cap g \cdot X_\alpha)$

For $|\lambda| + |\mu| + |\nu| = \dim X + nd$. Def $\langle X_\lambda, X_\mu, X_\nu \rangle_d$
$= \#\, C \subseteq X$ deg. d meeting

$g_1 X_\lambda$, $g_2 \cdot X_\mu$, $g_3 \cdot X_\nu$

Example $[X_\lambda] \cdot [X_\mu] = \sum_{|\nu| = |\lambda| + |\mu|} c_{\lambda\mu}^\nu [X_\nu]$

$c_{\lambda\mu}^\nu = \langle X_\lambda, X_\mu, X_{\nu^\vee} \rangle_0$



$QH(X) = H^*(X) \otimes \mathbb{Z}[q]$

$[X_\lambda] * [X_\mu] = \sum_{\nu, d \geq 0} \langle X_\lambda, X_\mu, X_{\nu^\vee} \rangle_d\, q^d [X_\nu]$.