Fact: $\mathrm{Hilb}(H^{(c)}, t) = \mathrm{Hilb}(\mathcal{H}^{(c)}, t) = \prod_{d=2}^{n} \frac{1-t^d}{1-t}$

Def f is c-quasiharmonic if $D_p^{(c)}(f) = 0$ $\forall p \in S^{<n}(V)^{S_n}$.

$Q\mathcal{H}^{(c)} = \{c\text{-quasiharmonics}\}$.

Lemma $\mathrm{Hilb}(Q\mathcal{H}^{(c)}) = \dfrac{(1-t^2)(1-t^3)\cdots(1-t^{n-1})}{(1-t)^{n-1}}$

$= \dfrac{\mathrm{Hilb}(\mathcal{H}^{(c)}, t)}{(1-t^n)}$

pf $(Q\mathcal{H}^{(c)})^* = S(V^*)/\langle e_2, e_3, \dots, e_{n-1}\rangle$
since the ideal has $\underbrace{\text{free}}_{\text{a natural}}$ resolution, it computes Hilb( ).

$\mathrm{Hilb}_U(\oplus\mathcal{H}^{(c)}, t)$

$= \sum_d [U \cdot \partial\mathcal{H}_d^{(c)}] \cdot t^d$

Cor from pf: $\mathrm{Hilb}_U(\oplus\mathcal{H}^{(c)}, t) = \dfrac{\mathrm{Hilb}_U(\mathcal{H}^{(c)}, t)}{1-t^n}$

if V = trival

$\mathrm{Hilb}((Q\mathcal{H}^{(c)})^{S_n}, t) = \dfrac{1}{1-t^n} = 1 + t^n + t^{2n} + t^{3n} + \cdots$

$\Rightarrow$ for each $k > 0$, $\exists!$ c-quasiharmonic S_n invariant $e_{kn}^{(c)}$ of degree $k \cdot n$

if $U = V$, simple $(n-1)$-dim S_n-module

$\mathrm{Hilb}_V(\mathcal{H}^{(c)}, t) = t + t^2 + \cdots + t^{n-1}$.