5.30

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Quasiharmonic polynomials for Coxeter groups and canonical elementary invariants

1. Motivation:

construct a "canonical" basis for Haiman's algebra H_n of diagonal coinvariants of S_n

$$H_n = \mathbb{C}\begin{bmatrix} x_1 & \cdots & x_n \\ y_1 & \cdots & y_n \end{bmatrix} \Big/ I$$

where

$$I = \left\langle \sum_{i=1}^{n} x_i^a y_i^b \right\rangle \quad \begin{matrix} a,b \geq 0 \\ a-b>0 \end{matrix}$$

$$\dim H_n = (n+1)^{n-1}.$$

THM (Berest, Etirca, Ginzburg [BEG], 02-03]

$$H_n = GR \; A_{\frac{n+1}{n}}$$

where $A_{\frac{n+1}{n}} = S(V)/\langle V_{n+1} \rangle$

here $V = \mathbb{C}^n / \mathbb{C}\langle (1,1,1,1) \rangle$

$V_{n+1} < S^{n+1}(V)$, $V_{n+1} \cong V$ as S_n-modules

$$V_{n+1} = \left\{ f \in S^{n+1}(V) : \nabla_y^{(c)} \Big|_{c=\frac{n+1}{n}} (f) = 0 \right\}$$

$$y \in V^*.$$

$$\nabla_y^{(c)}(f) = \partial_y(f) - c \sum_{i<j} \langle y, x_i - x_j \rangle \frac{f - (i,j)f}{x_i - x_j}$$

$f \in S(V)$, $y \in V^*$

Remark: $\left[\nabla_y^{(c)}, \nabla_{y'}^{(c)} \right] = 0$ commute