Special Stanford Algebraic Geometry/ Number Theory Seminar

CANONICAL SUBGROUPS IN p-ADIC FAMILIES OF ABELIAN VARIETIES

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Abstract

Motivated by problems in the theory of p-adic modular forms, it is important to study how p-power torsion subgroups "deform" in p-adic families of abelian varieties. A very fundamental construction, first worked out by Lubin (and Katz in the relative case) for elliptic curves, is the so-called "canonical subgroup". This was the foundation for Katz' theory of classical p-adic modular forms, and recent generalizations by several mathematicians have generalized this theory to the higher-dimensional case. However, each of these constructions suffers from one of two restrictions that can be unpleasant in practice: good reduction for all fibers, or the specification of discrete parameters (such as the degree of a polarization, so as to work on a fixed moduli space of finite type). Reflection on the matter suggests that such restrictions are artifacts of the constructions and should not be necessary.

I will explain an entirely different way to set up the higher-dimensional theory that avoids such restrictions, and is geometrically very natural (and consistent with other constructions). The essential tools that we use are the extremely pleasant topology and cohomology on Berkovich's theory of p-adic spaces, coupled with a theorem of Norman–Oort on the geometry of moduli spaces of abelian varieties in characteristic p (allowing p to divide the degree of a polarization). In the talk, we will treat Berkovich spaces like black magic.

Monday, May 2
4:15 p.m.
Room 381-U

`http://math.stanford.edu/~vakil/s0405/`