PROBLEM-SOLVING MASTERCLASS WEEK 1

1. Sum the series
$$\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{m^2 n}{3^m(n3^m+m3^n)}.$$
(Ryan Williams, 1999A4)

2. Find positive integers n and $a_1, \dots, a_n$ such that
$$a_1 + a_2 + \cdots + a_n = 1979$$
and the product $a_1 a_2 \cdots a_n$ is as large as possible. (Ravi Vakil, from the Red Rock Cafe in Mountain View, and problem A1 on some Putnam, I forget which year)

3. Define a sequence by $a_0 = 1$, together with the rules $a_{2n+1} = a_n$ and $a_{2n+2} = a_n + a_{n+1}$ for each integer $n \geq 0$. Prove that every positive rational number appears in the set
$$\left\{\frac{a_{n-1}}{a_n} : n \geq 1\right\} = \left\{\frac{1}{1}, \frac{1}{2}, \frac{2}{1}, \frac{1}{3}, \frac{3}{2}, \dots\right\}.$$
(Kiat Chuan Tan, 2002A5)

4. Let $x_1, x_2, \dots, x_{19}$ be positive integers each of which is less than or equal to 93. Let $y_1, y_2, \dots, y_{93}$ be positive integers each of which is less than or equal to 19. Prove that there exists a (nonempty) sum of some x_i's equal to a sum of some y_j's. (Daniel Le, 1993A4)

5. Evaluate
$$\int_0^1 \frac{\ln(x+1)}{x^2+1}\,dx.$$
(Ryan Williams, 2005A5)

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