PROBLEM-SOLVING MASTERCLASS WEEK 1

1. A game starts with four heaps of beans, containing 3, 4, 5 and 6 beans. The two players move alternately. A move consists of taking **either**

(a) one bean from a heap, provided at least two beans are left behind in that heap, **or**
(b) a complete heap of two or three beans.

The player who takes the last heap wins. To win the game, do you want to move first or second? Give a winning strategy. (Nathan Pflueger, 1995B5)

2. Consider integers $1, 2, 3, \ldots, 2n$ and pick more than n of them. Show that regardless of the choice made, one can find two integers picked such that one divides another. (Kiyoto Tamura, a problem from Erdos)

3. Let $S = \{1, 2, \ldots, n\}$ and let $S_1, S_2, \ldots, S_n$ be subsets of S satisfying $|S_i \cap S_j| \leq 1$ for $i \neq j$. Show that $\max(|S_1| + |S_2| + \cdots + |S_n|)$ is asymptotically $n\sqrt{n}$. (Bob Hough)

4. Evaluate

$$\int_0^2 \left(\left(x^3+1\right)^{1/2} + \left(x^2+2x\right)^{1/3} \right) \, dx.$$

(Natth Bejraburnin)

5. Evaluate

$$\sqrt[8]{2207 - \cfrac{1}{2207 - \cfrac{1}{2207 - \cdots}}}.$$

Express your answer in the form $\frac{a+b\sqrt{c}}{d}$, where a, b, c, d are integers. (Ravi Vakil, 1995B4)

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