MATH 113 PRACTICE MIDTERM

The actual midterm will have the same number of questions with the same instructions. They are of similar difficulty.

You may use only pens/pencils and scrap paper; calculators are not allowed (and also should not be useful), and this is a closed-book exam. The "A" problems just require answers, and no proofs or explanations. (Hint: they each have fast solutions, so don't dive into messy algebra.) They are each worth 2 points. For the "B" problems, justify your answers completely. They are each worth 6 points.

A1. Suppose $\vec{v}_1$, $\vec{v}_2$, $\vec{v}_3$, $\vec{v}_4$ are four vectors in $\mathbb{F}^5$. What are the possible values of

$$\dim \operatorname{span}(\vec{v}_1, \vec{v}_2, \vec{v}_3, \vec{v}_4)?$$

Give an example of each possibility.

A2. Suppose $\{\vec{v}\}$ is a linearly *dependent* set $\mathbb{F}^4$. Find $\vec{v}$.

A3. Find the column rank of

$$\begin{pmatrix} 9 & 1 & 3 & 2 & 0 & 8 & 0 \\ 1 & 2 & 0 & 9 & 0 & 3 & 9 \\ -2 & 2 & 0 & 9 & 12 & 9 & 2 \end{pmatrix}.$$

B1. Suppose $\vec{v}_1, \vec{v}_2, \dots, \vec{v}_7$ are vectors in a ten-dimensional vector space V. Show that they do not span V.

B2. Find a basis for the subspace

$$\operatorname{span}\left((1,2)\otimes(1,2), (1,1)\otimes(1,1), (1,0)\otimes(1,0), (0,1)\otimes(0,1)\right)$$

of $\mathbb{F}^2 \otimes \mathbb{F}^2$.

B3. Suppose W is a subspace of V, and V is a finite-dimensional vector space. Show that W and V/W are both finite-dimensional, and $\dim W + \dim V/W = \dim V$.

B4 (power series). *(Try this one only if you are done with the earlier problems.)* A *power series with coefficients in* $\mathbb{F}$ is something of the form $a_0 + a_1x + a_2x^2 + \cdots$ where $a_i \in \mathbb{F}$. (Unlike a polynomial, this sum isn't assumed to stop.)
(a) Show that the space of all power series (called $\mathbb{F}[[x]]$) is a vector space over $\mathbb{F}$.
(b) You can multiply power series by the following rule:

$$(a_0 + a_1x + a_2x^2 + \cdots)(b_0 + b_1x + b_2x^2 + \cdots)$$

$$= (a_0b_0) + (a_0b_1 + a_1b_0)x + (a_0b_2 + a_1b_1 + a_2b_0)x^2 + \cdots.$$

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If f is a power series, show that multiplication by f gives a linear transformation $\mathbb{F}[[x]] \to \mathbb{F}[[x]]$. For which f is this linear transformation invertible?

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