MATH 113 MIDTERM

You may use only pens/pencils and scrap paper; calculators are not allowed (and also should not be useful), and this is a closed-book exam. The "A" problems just require answers, and no proofs or explanations. (Hint: they each have fast solutions, so don't dive into messy algebra.) They are each worth 2 points. For the "B" problems, justify your answers completely. They are each worth 6 points.

A1. Suppose V_7, V_8, and V_9 are subspaces of a ten-dimensional subspace V, where $\dim V_i = i$. What are the possible dimensions of $V_7 \cap V_8 \cap V_9$?

A2. What is the (reduced) row echelon form of the following matrix?

$$\begin{pmatrix} 9 & 1 & -2 \\ 1 & 2 & 2 \\ 3 & 0 & 0 \\ 2 & 9 & 9 \\ 0 & 0 & 12 \\ 8 & 3 & 9 \\ 0 & 9 & 2 \end{pmatrix}$$

A3. Find $\dim(\mathcal{M}(m, n; \mathbb{F}) \otimes \mathcal{M}(p, q; \mathbb{F}))$. (Recall that $\mathcal{M}(a, b; \mathbb{F})$ is the vector space of $a \times b$ matrices.)

B1. Suppose $U = \operatorname{span}((1, 0, 0), (0, 1, 1))$ and $V = \operatorname{span}((1, 1, 0), (0, 0, 1))$. Give a basis for $U \cap V$. Give a basis for $U + V$.

B2. Suppose $\vec{v}_1, \ldots, \vec{v}_n$ are vectors in a vector space V. Show that $\dim \operatorname{span}(\vec{v}_1, \ldots, \vec{v}_n) \leq n$.

B3. Suppose

$$M = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}.$$

Let V be the subspace of the space $\mathcal{M}(3, 2; \mathbb{F})$ of 3×2 matrices defined by

$$V = \{N : MN = 0\}.$$

Show that V is a vector space. Find the dimension of this space.

B4. (The "Lights out" game). *(Try this one only if you are done with the earlier problems.)* Suppose $n \geq 2$ light bulbs are arranged in a row, numbered 1 through n. Under each bulb is a button. Pressing the button will change the state of the bulb above it (from on to off or vice versa), and will also change the neighbors' states. (Most bulbs have two

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neighbors, but the bulbs on the end have only one.) The bulbs start off randomly (some on and some off). For which n is it guaranteed to be possible that by flipping some switches, you can turn all the bulbs off? For part marks, do the cases $n = 3$, and $n = 4$. (Hint: this has something to do with the vector space $\mathbb{F}_2^n$.)