MATH 210 PROBLEM SET 6

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This problem set is due on Friday, March 9 at Jarod Alper's office door.

1. Prove that $(y^2 - x^3) \subset \mathbb{C}[x, y]$ is a radical ideal. (Hint: show it is prime.)

2. Describe all the algebraic subsets of $\mathbb{A}^1_{\mathbb{C}}$.

3. Show that the Zariski topology on $\mathbb{A}^2_{\mathbb{C}}$ is *not* the product topology on $\mathbb{A}^1_{\mathbb{C}} \times \mathbb{A}^1_{\mathbb{C}}$. (Possible hint: the "diagonal" $V(y - x)$ is a closed subset in the Zariski topology on $\mathbb{A}^2_{\mathbb{C}}$.)

4. Show that $\{(n, 0) | n \in \mathbb{Z}\} \subset \mathbb{A}^2_{\mathbb{C}}$ is not an algebraic subset.

5. Prove that $\sqrt{I}/I$ is the nilradical of R/I.

6. We've shown in class using the axiom of choice (in the guise of Zorn's lemma) that the radical of a proper ideal is the intersection of all prime ideals containing it. Show that this is true for Noetherian rings without using the axiom of choice.

Date: Friday, March 2, 2007.