MATH 121 PRACTICE MIDTERM

RAVI VAKIL

Justify all answers!

1. (a) Show that any homomorphism of fields is an inclusion.
(b) Show that $\mathbb{Q}(\sqrt{2})$ and $\mathbb{Q}(\sqrt{3})$ are not isomorphic fields.

2. Show that any characteristic 0 field K admits precisely one inclusion $\mathbb{Q} \to K$.

3. Lang p. 253 no. 7. Let E, F be two finite extensions of a field k, contained in a larger field K. Show that

$$[EF : k] \leq [E : k][F : k].$$

4. Show that the finite field $\mathbb{F}_{2^{100}}$ contains unique subfields isomorphic to $\mathbb{F}_8$ and $\mathbb{F}_{16}$. Find their intersection (in the form of $\mathbb{F}_q$ for some q).

5. Suppose E/F is an extension. Define the separable closure F^{sep} of F in E to be the separable elements of E/F. Show that F^{sep} is a subfield of E. If E/F is finite, show that E/F^{sep} is generated by a tower of pth roots. If E/F is algebraic, show that any element of E has some p^kth power in F^{sep}.

Date: Thursday, February 1, 2007. Problem 4 corrected February 3.