18.034 PROBLEM SET 5

Due March 17 in class. No lates will be accepted. Discussion is encouraged, with two caveats: (a) write up your solutions by yourself, and (b) give credit when others came up with ideas (you won't be penalized for this). Give explanations, not just answers. References are to Boyce and DiPrima; the answers to problems from the book are in the back of the book.

1. Problems 3.3.24, 3.3.25, 3.3.28, p. 145.
2. In this problem, we will find a differential equation of the form $y'' + p(t)y' + q(t)y = 0$, where $p(t)$ and $q(t)$ are continuous on the interval $-\infty < x < 1$, such that all solutions go to 0 as x approaches 1. In fact, these solutions will approach 0 to order 2 (i.e. they will approach the point $(1, 0)$ along the y-axis).
   (a) Find numbers a and b so that $t^2y'' + aty' + by = 0$ has solutions x^2 and x^3 (on the interval $0 < x < \infty$). Find (with proof) *all* solutions to this differential equation on this interval. What can you say about the solutions as they approach 0 from the right?
   (b) Now find a differential equation of the desired form that has solutions $(1 - t)^2$ and $(1 - t)^3$ (on the interval $-\infty < x < 1$). Find (with proof) *all* solutions to this differential equation on this interval. Sketch several of the solutions.
3. *Using the Wronskian to find all solutions to a homogeneous equation knowing only one.* Consider the differential equation $ty'' - (1 + t)y' + y = 0$ on the interval $t > 0$. Notice that one solution is of the form $y_1 = 1 + t$. Suppose y_2 is another solution, such that y_1 and y_2 are linearly independent. Find the possible functions $W(y_1, y_2)$. Use this to find one such y_2. Find all solutions to this differential equation. (Check your answer!)
4. *Using the Wronskian to find solutions to a nonhomogeneous equation knowing the homogeneous solutions: Convolutions Take One.* Problems 3.7.22, 3.7.24–27, p. 177–178. (Convolutions will turn up again later in the course.) In 3.7.27, write down the function K explicitly in each of the 3 cases in a single table.
5. *Generalizing Abel's theorem to higher order.* Problems 4.1.20 and Problem 4.1.23, p. 207–208.
6. Problem 4.1.25, p. 208.

Date: March 13, 2000.