18.024 PRACTICE QUIZ III

Solutions will be discussed in recitation on Thursday. Sketches of solutions will also appear on the web page on Thursday.

1. (16 points) Let C be the curve $\vec{\alpha}(t) = (t^2, 2t, -3t)$ from $(0,0,0)$ to $(1,2,-3)$ in $\mathbb{R}^3$.

(a) Evaluate $\int_C \vec{F} \cdot d\vec{\alpha}$ if $F(x,y,z) = 3x\vec{i} + xy\vec{j} + yz\vec{k}$.
(b) Evaluate $\int_C \vec{\nabla}\phi \cdot d\vec{\alpha}$ if $\phi(x,y,z) = xy^2 \sin z$.

2. (16 points) For each of the following vector fields, either find a function ϕ such that $\vec{\nabla}\phi = \vec{F}$ or explain how you know that no such function exists.

(a) $\vec{F}(x,y,z) = (y^2, 2xy+2, yz)$
(b) $\vec{F}(x,y,z) = (y^2, 2xy+2, z)$
(c) $\vec{F}(x,y,z) = (y^2, 2xy+2, xz)$

3. (16 points) Set up a triple integral for the volume of the solid consisting of those points for which $x \geq 0$, $y \geq 0$, $z \geq 0$, and $x + y^2 + z \leq 1$,

(a) in which the first integration (the one on the inside) is with respect to z. Your answer should be of the form
$$\int_?^? \int_?^? \int_?^? 1\, dz\, dy\, dx \quad \text{or} \quad \int_?^? \int_?^? \int_?^? 1\, dz\, dx\, dy.$$
(b) in which the first integration is with respect to y.

4. (16 points) Find the y-coordinate $\overline{y}$ of the centroid of the region in the plane bounded by $y = x^4$ and $y = 1$.

5. (16 points) $\vec{f}$ be a continuously differentiable vector field defined on an open set U in V_m. Consider the following conditions on $\vec{f}$:

(a) $\int_C \vec{f} \cdot d\vec{\alpha} = 0$ for every closed piecewise-smooth curve C in U.
(b) $\vec{f} = \vec{\nabla}\phi$ for some function ϕ defined on U
(c) $D_i f_j = D_j f_i$ in U (where $\vec{f}(\vec{x}) = (f_1(\vec{x}), \ldots, f_n(\vec{x}))$.

- Does (a) imply (b)? **Y/N**
- Does (b) imply (a)? **Y/N**

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- Does (b) imply (c)? **Y/N**
- Does (c) imply (b)? **Y/N**

(+4 for each correct answer, −4 for each incorrect answer)

6. (20 points) Suppose $\vec{\alpha}_1$, $\vec{\alpha}_2$, $\vec{\alpha}_3$ are paths $[0,1] \to \mathbb{R}^2$ given by

$$\vec{\alpha}_1(t) = (0, 1-t) \qquad \text{for } 0 \le t \le 1,$$

$$\vec{\alpha}_2(t) = \begin{cases} (0, 2t) & \text{for } 0 \le t \le 1/2, \\ (2(t-1/2), 1) & \text{for } 1/2 \le t \le 1, \end{cases}$$

$$\vec{\alpha}_3(t) = \begin{cases} (2t, 1) & \text{for } 0 \le t \le 1/2, \\ (2-2t, 2-2t) & \text{for } 1/2 \le t \le 1. \end{cases}$$

Suppose $\vec{F}$ is a vector field on $\mathbb{R}^2$, and $\int \vec{F} \cdot d\vec{\alpha_1} = 3$, $\int \vec{F} \cdot d\vec{\alpha_2} = e$, and $\int \vec{F} \cdot d\vec{\alpha_3} = \pi$.

(a) Suppose the path $\vec{\alpha}_4 : [0, \pi/2] \to \mathbb{R}^2$ is given by $\alpha_4(t) = (\sin t, \sin t)$. Calculate $\int \vec{F} \cdot d\vec{\alpha_4}$.

(b) Is $\vec{F}$ conservative? (Explain.)

(Hint: the paths are sketched below.)