

18.024 PRACTICE QUIZ I

1. (20 points) Let L_1 be the line through the point $P = (a, 0, 0)$ on the x -axis with direction vector $(-3, 1, -1)$. Let L_2 be the line $X = (1, 2, 0) + t(1, -1, 2)$. If L_1 and L_2 intersect, find the point P .

2. (24 points) Let A be a k by n matrix; let r be the rank of A . Answer the following questions in terms of n , k , and r . (Give answers only.)

- What can you say about the dimension of the row space of A ?
- What can you say about the dimension of the solution space of the equation $AX = 0$?
- What can you say if the system $AX = C$ fails to have a solution for some C ?
- What can you say if you know A has an inverse?

3. (20 points) Find conditions on a , b , c that are both necessary and sufficient for the following system to have a solution.

$$\begin{array}{rcl} y & - & 2z = a \\ x & - & y + z = b \\ x & + & y - 3z = c \end{array}$$

4. (20 points) Find the inverse of the matrix

$$A = \begin{pmatrix} 2 & 0 & 0 & 1 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix}.$$

5. (16 points) Let A be a 5 by 5 matrix. Show that if A^3 has rank less than 5, then A has rank less than 5.

Another tricky question: Suppose A , B , and C are three vectors in V_5 . Can $3A+2B+4C$, $A+4B+2C$, $9A+4B+3C$, and $A+2B+5C$ be linearly independent?