

Math 63CM Homework # 8

Due in section on Friday, March 13.

1. Problem 5.2 in Brendle.
2. Problem 5.3 in Brendle.
3. Problem 5.4 in Brendle.
4. Problem 5.5 in Brendle.
5. Consider the equation

$$(1) \quad cu' = u'' + u(1 - u), \quad -\infty < x < +\infty,$$

with some $c > 0$ and write it as a first order system

$$u' = -v, \quad v' = cv + u(1 - u).$$

- (i) Show that the only equilibria are $u = 0, v = 0$ and $u = 1, v = 0$.
- (ii) Show that if $c \geq 2$ then there exists an invariant region D in the (u, v) plane such that D is a triangle with the vertices $(0, 0)$, $(1, 0)$, and (a, b) with $a > 0$ and $b > 0$ (find such a, b explicitly).
- (iii) Show that if $c \geq 2$ and $(u(0), v(0))$ lies on the stable manifold of $(1, 0)$, then $u(x) \rightarrow 0$ as $x \rightarrow -\infty$ and $v(x) \rightarrow 0$ as $x \rightarrow -\infty$. Deduce that for any $c \geq 0$ there is a solution $u(x)$ to (1) such that $u(x) \rightarrow 0$ as $x \rightarrow -\infty$ and $u(x) \rightarrow 1$ as $x \rightarrow +\infty$. Show that this solution satisfies $0 < u(x) < 1$ for all $x \in \mathbb{R}$.
- (iv) Show that if $|c| < 2$ there is no such solution to (1).