

Math 63CM Homework # 7

Due in section on Friday, March 6.

1. Problem 4.1 in Brendle.

2. Problem 4.2 in Brendle. As a clarification, a sink is an equilibrium x_0 for an ODE $x'(t) = F(x)$ such that $F(x_0) = 0$ and all eigenvalues of the matrix $A = DF(0)$ have negative real part, a source is an equilibrium point such that all eigenvalues of A have a positive real part, and a saddle is an equilibrium point such that some eigenvalues of A have a positive real part, some have a negative real part, and none have a zero real part.

3. Problem 4.3 in Brendle.

4. (i) Consider a 2×2 system of ODEs

$$\begin{aligned}x'(t) &= y(t), \\y'(t) &= -x(t) - x^3(t).\end{aligned}$$

Show that $(0, 0)$ is a stable equilibrium point. Is this a Hamiltonian system? Do solutions to this system exist for all $t > 0$ for any initial condition (x_0, y_0) ?

(ii) Consider a 2×2 system of ODEs

$$\begin{aligned}x'(t) &= y(t), \\y'(t) &= -x(t) + x^3(t).\end{aligned}$$

Is this a Hamiltonian system? Do solutions to this system exist for all $t > 0$ for any initial condition (x_0, y_0) ? Why is there no contradiction?

5. Problem 5.1 in Brendle.