Adaptive evolution and concentrations in nonlocal parabolic PDEs

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Abstract

Living systems are subject to constant evolution through the three darwinian processes of population growth, selection and mutations. In a very simple, general and idealized description, their environment can be considered as a nutrient shared by all the population. This alllows certain individuals, characterized by a 'phenotypical trait', to proliferate faster because they are better adapted to use the environment. This leads to select the 'fittest trait' in the population (singular point of the system). On the other hand, the new-born individuals can undergo small variations of the trait under the effect of genetic mutations. In these circumstances, is it possible to describe the dynamical evolution of the current trait? A new area of population biology that aims at describing mathematically these processes is born in the 1980's under the name of 'adaptive dynamics' and, compared to population genetics, considers usually asexual reproduction, a continuous phenotypical trait and population growth.

We will give a self-contained mathematical model of such dynamics, based on parabolic equations, and show that an asymptotic method allows us to formalize precisely the concepts of monomorphic or polymorphic population and describe the evolution of the 'fittest trait'. Mathematically, the interest comes from concentration effects after an appropriate rescaling. The solution converges to a (sum of) Dirac mass(es) supported on a hypersurface that results from the nonlinearity. The difficulty is to evaluate the weight and position of the moving Dirac mass(es) that desribe the population. We will show that a new type of Hamilton-Jacobi equation, with constraints, naturally describes this asymptotic. Some additional theoretical questions as uniqueness for the limiting H.-J. equation will also be addressed.

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Course 1. Population models of adaptive evolution.

Course 2. The basic BV estimate.

Course 3. Dirac concentrations and the constrained Hamilton-Jacobi equations

Course 4. The smooth case and dynamics on the hypersurface

Course 5. Related extensions: Periodic environment or traveling waves or resistance to therapy in cancer

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