

**ERRATA FOR:  
HOLOMORPHIC  $K$  - THEORY, ALGEBRAIC CO-CYCLES, AND LOOP  
GROUPS**

RALPH L. COHEN AND PAULO LIMA-FILHO

It was pointed out to the authors by Eric Friedlander that there are problems with the proof of theorem 17 in section 3 of [1] which states that the map

$$(0.1) \quad \beta : K_{hol}(\prod_n \mathbb{P}^1) \rightarrow K_{top}(\prod_n \mathbb{P}^1)$$

is a  $\Sigma_n$  - equivariant homotopy equivalence. Here  $\beta$  is the natural map from holomorphic  $K$  - theory to topological  $K$  - theory discussed earlier in the paper.

While we still conjecture that theorem 17 is true, these problems mean that at present there is no proof. Similarly the corollaries drawn from this theorem in section 3 as well as the results leading up to its proof (corollaries 18, 19, and 20, theorem 23) must now be viewed as not proven.

The main problem in the argument occurs on p. 363 where it is stated that the map

$$(0.2) \quad \beta : \prod_{\theta} K_{hol}^{\sim}((\mathbb{P}^1)^{(\theta)}) \rightarrow \prod_{\theta} K_{top}^{\sim}((\mathbb{P}^1)^{(\theta)})$$

is a  $\Sigma_n$ -homotopy equivalence. While it is true that each individual map  $K_{hol}^{\sim}((\mathbb{P}^1)^{(\theta)}) \rightarrow K_{top}^{\sim}((\mathbb{P}^1)^{(\theta)})$  is a homotopy equivalence, and that the map (0.2) is equivariant, the argument only used the  $\Sigma_n$ -action permuting the factors in the product, neglecting the action on the space level. At present we do not know a proof that the map is an equivariant equivalence with the full action of the symmetric group.

We remark that the results in sections 1 and 2 of the paper are unaffected by the gap in the proof of theorem 17 and so remain valid. The main consequence of theorem 17 in the paper was that the Chern character homomorphism for holomorphic  $K$  - theory is a rational isomorphism (theorem 1 of the paper). The proof of this fact in the paper therefore has a gap. However E. Friedlander and M. Walker have recently announced a proof using different techniques that a multiplicative Chern character is a rational isomorphism for

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smooth quasiprojective varieties. With the validity of this result, the consequences derived in section 5 of the paper remain valid.

The following are corrections or additions to sections 4 of the paper.

1. Insert on p. 363 between lines -1 and -2:

Let  $X_1 \subset X_2 \subset \cdots \subset X_i \subset \cdots$  be a direct system of inclusions of projective varieties. Let  $X = \cup_i X_i$  be the colimit. Define the holomorphic  $K$ -theory of  $X$  to be the homotopy inverse limit of the holomorphic  $K$ -theories of the  $X_i$ 's:

$$K_{hol}(X) = \mathop{\text{hol}}\limits_{\leftarrow} \text{lim} K_{hol}(X_i).$$

2. Replace all the direct sums in this section with direct products. Similarly do so in the statement of theorem 1.
3. On p. 364 remove the “end proof box” prior to the statement of lemma 28.

#### REFERENCES

- [1] R.L. Cohen and P. Lima-Filho, *Holomorphic  $K$ -theory, algebraic cocycles, and loop groups*,  $K$ -theory, **23**, (2001), 345-376.
- [2] E.M. Friedlander and M.E. Walker, *Rational isomorphisms between  $K$ -theories and cohomology theories*, to appear.

DEPT. OF MATHEMATICS, STANFORD UNIVERSITY, STANFORD, CALIFORNIA 94305

*E-mail address*, Cohen: `ralph@math.stanford.edu`

DEPARTMENT OF MATHEMATICS, TEXAS A&M UNIVERSITY, COLLEGE STATION, TEXAS

*E-mail address*, Lima-Filho: `plfilho@math.tamu.edu`