Central values of automorphic $L$-functions as Fourier coefficients (Maass Case)

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In this expository talk, I will discuss a beautiful formula of Katok and Sarnak (1993) which gives the sum of a Maass cusp form $\varphi$ over Heegner points (resp. cycles) in terms of the fourier coefficients of a corresponding half-integral weight Maass form. In the special case $d = 1$, the sum over Heegner points reduces to simple factors times $L(1/2, \varphi)$. Thus, the K-S formula gives the natural generalization of the previous week’s talk to the non-holomorphic case. The proof of the formula makes explicit a type of “Shimura lift” for non-holomorphic forms developed by Niwa, a mapping from a weight $1/2$ space to a weight 0 space. To sketch the proof of this Shimura lift, we will have to use Siegel’s $\Theta$-function, the Weil representation, the Casimir operator, class groups and lots of other fun stuff. In the end, the K-S formula turns out to be a kind of trace formula. Details will be included only as time permits. My goal is to make this talk depend on the previous for motivation only, and I may or may not succeed.