Math 52H: Homework N7

Due to Friday, March 2

1. a) Prove that any path connected subset of $\mathbb{R}^n$ is connected.

b) Prove that for open subsets of $\mathbb{R}^n$ the notions of connectedness and path-connectedness coincide, i.e. any open connected set $U \subset \mathbb{R}^n$ is path connected.

c) Prove that for any subsets of $\mathbb{R}$ the notions of connectedness and path-connectedness coincide, i.e. any connected set $A \subset \mathbb{R}$ is path connected. Show that if $A \subset \mathbb{R}$ is open and connected then A coincides with the interval (a, b), where we allow $a = -\infty$ and $b = \infty$.

2.* Prove that any closed connected (Hausdorff) 1-dimensional manifold is diffeomorphic to the unit circle $S^1 = \{x_1^2 + x_2^2 = 1\} \subset \mathbb{R}^2$.

3. Prove that the submanifold $T = \{x_1^2 + x_2^2 = 1,\ x_3^2 + x_4^2 = 1\} \subset \mathbb{R}^4$ and the submanifold $U \subset \mathbb{R}^3$ given in cylindrical coordinates (r, ϕ, z) by the equation $(r-2)^2 + z^2 = 1$, are diffeomorphic.

4. Compute the integral

$$\iint\limits_D \left| \frac{x+y}{2} - x^2 - y^2 \right| dxdy,$$

where D is the unit disc $\{x^2 + y^2 \leq 1\}$.

5. Compute the volume of the domain $U \subset \mathbb{R}^3$ defined by

$$U = \{x \geq 0, y \geq 0, 1 \leq xy \leq 2, x \leq y \leq 2x, 0 \leq z \leq x + y\}.$$

Each (sub-)problem is 10 points. Problem 2 is more difficult and it is an extra-credit.