Math 52H: Homework N6

Due to Friday, February 24

1. Use Fubini's theorem to compute the following multiple integrals:

a) $\int\limits_{D}(x-y)dA$, where D is a triangle with vertices $(0,0),(1,0)$, and $(2,1)$.

In the following problems B is the domain in $\mathbb{R}^3$ defined by

$$B=\{0\le x\le 1, 0\le y\le 1, 0\le z\le xy\}.$$

b) $\int\limits_{B} xdV$,

c) $\int\limits_{B} ydV$,

d) $\int\limits_{B} zdV$

e) $\int\limits_{B} xydV$.

2. Use change of variables formula (together with Fubini's theorem) to compute the following integrals.

a) $\int\limits_{S}(x^2+y^2)dV$, where $S=\{x^2+y^2\le 2x\}$.

b) $\int\limits_{S}\sqrt{1-\frac{x^2}{a^2}-\frac{y^2}{b^2}}dV$, where $S=\{\frac{x^2}{a^2}+\frac{y^2}{b^2}\le 1\}$.

3. Find the area of a curvilinear quadrangle bounded by the arcs of the parabolas

$$x^2 = ay, x^2 = by, y^2 = \alpha x, y^2 = \beta x, \quad \text{where} \quad o < a < b,\ 0 < \alpha < \beta.$$

Hint: introduce new variables (u, v) such that $x^2 = uy, y^2 = vx$.

4. In what ratio does the hyperboloid $\{x^2 + y^2 - z^2 = a^2\}$ divide the volume of the ball $\{x^2 + y^2 + z^2 \leq 3a^2\}$.

5. Solve Exercise 8.22.2 from the online text:

Prove that if A is nowhere dense then either $\mathrm{Vol} A = 0$, or A is not measurable in the sense of Riemann. Find an example of a non-measurable nowhere dense set. (A set A is called *nowhere dense* if $\mathrm{Int} A = \varnothing$.)

Each subproblem of 1 is 5 points. All other problems and subproblems are 10 points each.