Math 177: Homework N2

Due on Wednesday, April 27

1. Consider a differential equation

$$2xy' - y = \ln y'.$$

Find all the solutions and the discriminant.

2. Find a curve on the plane whose tangent lines form with the coordinate axis triangles of area $2a^2$

3. Prove that the rank of any skew-symmetric bilinear form is even.

4. Let us view $\mathbb{C}^n$ as $\mathbb{R}^{2n}$ with the operator of maltiplication by i. Denote by

- $GL(2n)$ the group of invertible real linear transformations of $\mathbb{R}^{2n}$;
- $GL(n,\mathbb{C}) \subset GL(2n,\mathbb{R})$ the group of invertible complex linear transformation of $\mathbb{C}^n$;
- $U(n) \subset GL(n,\mathbb{C})$ the group of unitary transformations of $\mathbb{C}^n$, i.e. transformations preserving the Hermitian form $\sum_1^n z_k \overline{w_k}$;
- $Sp(2n) \subset GL(2n,\mathbb{R})$ the group of symplectic transformations of $\mathbb{R}^{2n}$, i.e. transformations preserving the symplectic form $\omega = \sum_1^{2n} x_k \wedge y_k$;
- $SO(2n) \subset GL(2n,\mathbb{R})$ the group of orthogonal transformations of $\mathbb{R}^{2n}$, i.e. transformations which preserves the standard scalar dot-product.

Prove that

$$SO(2n) \cap Sp(2n) = GL(n, \mathbb{C}) \cap SO(2n) = GL(n, \mathbb{C}) \cap Sp(2n) = U(n).$$

5. Prove that the plane fields given by Pfaffian equations $dz - ydx = 0$ and $dz - \frac{1}{2}(xdy - ydx) = 0$ are diffeomorphic, but the plane fields $dz - \frac{1}{2}(xdy - ydx) = 0$ and $dz - \frac{1}{2}(xdy + ydx) = 0$ are not.

6. Consider a PDE

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = u - xy.$$

Solve the Cauchy problem for the initial data $u(2, y) = 1 + y^2$.

Each problem is 10 points.