

Math 177: Homework N1

Due on Wednesday, April 15

1. Consider a system of two quasi-homogeneous equations

$$\begin{aligned}xy \frac{dy}{dx} &= z^2, \\ z \frac{dz}{dx} &= xy, \quad x, y, z > 0.\end{aligned}$$

Find a change of variables which reduces it to a first order system.

2. (*Last Exercise in Section 1.C in Arnold's book.*) It is known that a top with a vertical axis has a critical angular velocity: if the angular velocity is greater than critical then the top stands up firmly vertically, and if it less, it fails. How does the critical velocity change if we take the top to the moon, where the gravitation force is 6 times less than on the earth?
3. Consider a pendulum $\ddot{x} = -\sin x$. Find the limit of its period when the amplitude goes to 0.
4. Find the solutions, the criminant and the discriminant of the equation

$$(y')^3 + 3xy' - 3y = 0.$$

5. Let $f = \sum_1^n a_{ij}x_i x_j$ be a quadratic form on \mathbb{R}^n . Show that its Legendre transform $g(p)$ is again a quadratic form $g(p) = \sum_1^n b_{ij}p_i p_j$ and the value of these forms and the points corresponding to each other under the Legendre map coincide.

(Legendre map is a diffeomorphism $x \mapsto p$ which is used in the definition of Legendre transform.)

Each problem is 10 points.